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"On the Application of Maxwell's Curves to Three-colour Work, with Especial Reference to the Nature of the Inks to be employed, and to the Determination of the Suitable Light-filters." By REGINALD S. CLAY, B.A., D.Sc. Communicated by Sir W. DE W. ABNEY, K.C.B., F.R.S. Received April 25, —Read June 20, 1901.

PART I.—THEORETICAL.

1. *Three-colour Projection.*

Maxwell showed that any colour in the spectrum could be matched by a suitable mixture of three monochromatic lights, red, green, and violet, and his celebrated curves give at each part of the spectrum the intensity of these lights necessary to imitate the colour at that point both in hue and luminosity. Other experimenters have since repeated his measurements with improved apparatus; and throughout this essay I shall employ the curves found by Sir Wm. Abney as being probably the most accurate.†

Thus it is possible to photograph a spectrum in three colours only. Three negatives must be taken through "filters" which allow the colours to pass respectively in amounts determined by the above curves. One filter must allow light to pass according to the red curve, *i.e.*, the extreme red is just transmitted, and the bright red fully passed. Then the filter absorbs the yellow slightly, and the absorption increases along the spectrum, until in the blue and violet it is nearly complete. From this negative a transparency is made, and projected with red (monochromatic) light on a white screen. So for the green and violet. Assuming this done, we have a spectrum illuminated everywhere with the three monochromatic lights in the proportions indicated by the curves. Thus the spectrum will be reproduced exactly. (See Notes 1 and 2.)

If, instead of illuminating the three transparencies with monochromatic lights, we use red, green, and violet lights produced by passing white light through coloured glasses (providing that these lights are of the same dominant hue as the primary colour sensations), we shall still obtain a spectrum that appears almost correct. The light obtained by a coloured glass is not monochromatic, but, if the hue is

† "The Colour Sensations in Terms of Luminosity," 'Phil. Trans.,' 1899.

correct, it may be matched by a mixture of the monochromatic light and white. Thus the colours on the screen will not be true spectrum colours, but spectrum colours diluted with white.

The amount of white, of course, depends upon the glasses used, and in matching some parts of the spectrum there will be more white than in others. Yet the result is very good; indeed, without direct comparison the white would hardly be noticed. The beautiful pictures produced and exhibited with such coloured glasses by Mr. Ives is a very good proof of the fact that an admixture of white is of very small consequence.

Note 1.—The above is not strictly accurate, as it is not possible to match some parts of the spectrum even when using pure spectrum colours. The match can only be made after white has been added. The reason for this can be seen from the curves. Owing to the overlapping of the sensation curves, the green sensation is nowhere excited alone, but is always accompanied by a small excitation of the red and violet. Thus the spectrum green—although really a pure colour—excites all three sets of sensations in the eye, *i.e.*, it produces the sensation green and white. Now, to match the yellow, we require to excite only green and red, and this we cannot do using spectrum red and green, for the green always excites to some extent the violet sensation. This violet combines with proper proportions of green and red to produce white, so that we can imitate the yellow when it is mixed with a little white, but not the pure yellow alone. The same is true of the other end of the spectrum where the red sensation should only be slightly excited.

We shall in what follows neglect this necessity for the addition of white in speaking of colour matches as it only complicates the question without materially affecting the results. (Where my statements require modification in consequence I shall indicate it by a star (*).)

Note 2.—In Abney's paper above referred to, he shows that the third sensation is probably not violet, but a blue, which is near the blue lithium line in the spectrum. As I shall be using the word "blue" in speaking of the double colour blue ink, to avoid confusion, I shall throughout refer to the third colour sensation as "violet." I shall, however, use Abney's curves, and by the term "violet" shall mean his "blue sensation."

2. *Three-colour Printing.*

When we turn to printing, matters are not so simple. One's first impulse would be to select those parts of the original picture in which red light occurs, or which excite the red sensation, and print them with a red ink, and so with the green and violet; but a little consideration will show that this would not be right. For instance, a yellow object affects both our red and green sensations, according to Maxwell's

curves, and thus we should have to print it with the red and green inks. Now the original yellow was equivalent to the addition of a red and green light, so that it ought to be brighter when both colours are there than when only one. The reverse would be the case if printed as above, and, of course, as a white object reflects all the colours, it would be printed with all three inks. Now, though red, green, and violet lights when added give white, red, green, and violet inks superposed on white paper certainly will not make white.

In fact, inks produce colour by absorption. Thus, instead of successive inks adding to the light, they each reduce it.

Suppose, then, we proceed the other way, and instead of looking for those parts of the picture where red is reflected and printing with red ink, we print those parts where no red is reflected with an ink which absorbs red, but reflects all the other colours. This will be a bright bluish-green ink. In the same way we will print those parts which reflect no green with an ink which absorbs that colour; this will be a bright magenta or almost a pink. And those parts which reflect no violet we will print with a yellow ink. Now a yellow object reflects red and green but no violet. Hence it will have to be printed with the violet absorption ink only—that is, the yellow—and with neither of the others. A red object reflects neither green nor violet, and will be printed with the pink and yellow inks, which will leave red only. As a yellow object is printed with only the one ink, but a red one with two inks, the yellow will be the brighter, and this is as it should be.

As the inks are to absorb red, green, and violet respectively, they will, roughly speaking, be complementary to those colours, that is, when added to them either by a double-image prism, or by reflection from a clear glass surface, they should give white. Still, complementary is a very vague description of a colour, and it must be our aim presently to define it more precisely.

The above theory is due to Mr. Ives.

3. *Application of Maxwell's Curves.*

As we said at first, these curves were derived by adding three *monochromatic* lights, and are only strictly true of such colours. If, however, we use lights compounded of—or, at least, capable of being matched by—one of these monochromatic colours and white, we shall still obtain a result very nearly as perfect as with the pure colours. It can easily be seen, however, that if any other colour than white were combined with the pure colours the result would be spoilt.

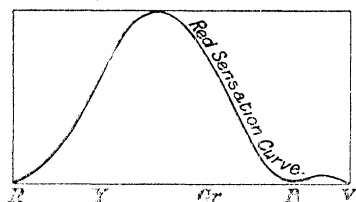
All colours, including white, can be matched both in hue and luminosity* by combining the three monochromatic colours, red, green, and violet, in proportions given by Maxwell's curves. It is easy to

* See Note 1, p. 27.

reproduce them by adding lights; is it possible to do so by printing with inks? It is, at least, theoretically. The result, however, would be a very dark picture of no use in practice, as it would require to be looked at either in full sunlight or some equally powerful light.

Suppose, then, we take a card and first print it all over with an ink that absorbs all the spectrum except a narrow band in the red, another narrow band in the green, and a third in the violet (whether such an ink could be found is immaterial as our aim is purely theoretical). If the width and position of these bands are correct, *i.e.*, if the colours left match the three sensation colours in hue, and their luminosities are properly proportioned, the card will now appear a neutral grey. In a powerful light it will appear white. This is to be our white.

The inks must each absorb one of these bands of colour and leave the other two bands unaffected. The yellow ink will absorb the violet, the pink ink the green, and the blue ink the red band.



Now if we print with these inks in amounts which are the complementaries of the amounts indicated by Maxwell's curves, we shall be transmitting lights in amounts which are the same as those given by Maxwell's curves, and thus we shall exactly match all the colours.† A picture so obtained will be correct both in hue and luminosity,* but it will have to be examined in a very powerful light.

The three bands of colour left, referred to above, ought all to be very narrow so that the light may be approximately monochromatic.

But now suppose these bands not to be infinitely narrow. The light will no longer be monochromatic and a single band will excite more than one colour-sensation in our eyes. For instance, a band in the red—unless in the very extreme red—excites both the red and green sensations; a band in the green excites all three (the red and violet will be nearly in the proportion to form white, and, if desired, the colour may be made equivalent to green and white by a proper selection of the width and position of the band); a band in the violet—unless in the extreme violet—will excite also red and green

† The curves referred to above are those in which equal heights of ordinate form white, *e.g.*, Abney, *loc. cit.*, p. 284, fig. 8.

* See Note 1, p. 27.

with generally the green in excess of that required to combine with the red to form white. Thus with such bands for our primary colours, we should have not the true sensation colours, but those colours with an addition of either green or white. The white will not matter much, but the green will, of course, spoil the effect.

With these bands more light is used, and both the grey produced by the original printing and the final colours will be much brighter; thus the picture could be examined in a less powerful light.

Lastly, imagine the bands still wider; in fact, allow them just to touch, so that they divide up the whole spectrum between them; and suppose each ink to exactly absorb one of these bands. The first printing in neutral colour is no longer necessary, and we shall see the full white of the card. Also the picture will no longer require a strong illumination. But we shall now have our primary colours mixed with large amounts of other colours. The red will be mixed with green, the violet will be diluted with green and red (see Abney's curve, *loc. cit.*, p. 283), and the green with both red and green in proportions which may or may not form white.

Note.—It is possible to arrange that the red and violet in the middle band shall make white, but it is not possible to so arrange that the red band shall be diluted with white, for there is no violet at the red end of the spectrum. Thus the blue ink cannot be made the complementary of the red sensation if it is to leave a red band of any reasonable width, unless it has an absorption at the violet end of the spectrum, and this introduces a very serious fault, as the same colour would then be absorbed by more than one ink. (See 6.)

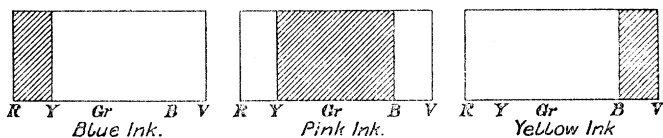
It will now be obvious that if we print these inks in amounts the complementaries of the Maxwell's curves (as in the theoretical case above) the colours we produce will be far from correct. The red will be diluted with green and so on.

If the red could be diluted with white instead of green the picture would be much improved; in fact, it would be almost as good as if the colours were pure. And fortunately this is quite possible, not however by modifying the inks but by altering the proportions in which they are printed, *i.e.*, by varying the curves to suit them.

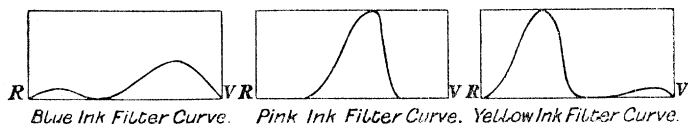
Suppose, for instance, we want to print a colour to match the red sensation. If we print with the green and the violet absorption inks—the pink and yellow ones—we shall leave the red end of the spectrum, which the curves show to excite the red and green sensations. We want the red only, or—if we cannot obtain that—we must be content with red and *white*. This we can obtain with our inks by printing the violet absorption ink—the yellow—rather less fully. The violet and green then, with a part of the red will form white, and leave red only. So with any colour, if we cannot obtain it pure, we can always match it when diluted with white. The amount of white

so formed can be increased in any part of the spectrum by printing all three inks less fully, and the best result will be obtained when the added white is everywhere proportional through the spectrum to the luminosity of the colour.

The three inks we have been discussing are supposed to absorb the spectrum thus :



The curves to be used in printing these must be adjusted so that the spectrum may be matched in luminosity as well as hue, the added white being itself (roughly) proportional to the luminosity. To adjust the luminosity of the colours, it will be observed that the colours which are relatively too bright can be reduced by printing more fully with all three inks. The curves are roughly



I have calculated the exact curves for such inks to enable them to match exactly the colour sensations as determined by Abney, and the results are given in the second part of this paper.

4. The Effect of the Addition of White.

There are several ways in which this can be shown not to be important when the amount of white is not very great.

In the first place, we are accustomed to use colours which are diluted with white. According to Abney the light reflected from vermilion contains 2·5 per cent. of white, from cobalt 55 per cent., from French ultramarine blue 61 per cent., and from chrome yellow 26 per cent. of white (p. 166, 'Colour Measurement'); whilst in the same way the light transmitted by a ruby glass contains 2 per cent. of white, by a canary glass 26 per cent. of white, by a green glass 31 to 61 per cent., by a cobalt glass 42 per cent. of white. That is to say, the transmitted light from a cobalt glass can be matched by 58 per cent. of a spectrum colour with 42 per cent. of white.

Then again about 3 per cent. of white can be added to orange, rather less to green, and so less along the spectrum till 0·8 per cent. in

the blue-green can be added *without being perceived* even when the diluted colour is compared directly with the pure (p. 132, 'Colour Measurement'). It can thus easily be believed that much more can be added without appreciable effect when no direct comparison is made with the pure colours.

But probably the best example of the unimportance of the addition of white is given by Mr. Ives's results, where the colours used as primary colours are produced by transmitting white through a red, green, and violet glass respectively; when, as in the examples above, there is transmitted a light which is equivalent to a pure colour with a percentage of white, which in some cases is quite considerable. I confess my surprise at the excellence of the results obtained, which show how exceedingly bad a judge the eye must be of an addition of white to a colour. Provided the quantity that will be left in the printing is not greater than the quantities he obtained by the transmission through his glasses—and I see no reason why inks should not be produced to secure this—the pictures obtainable by the three-colour processes ought not to be inferior to those he obtains by the superposition of the three-coloured transparencies.

We can realise the effect of this addition of white by supposing a coloured picture projected on a screen in a room that is not quite dark. The screen will then reflect, in addition to the colours projected, a certain amount of the diffused light of the room. As this coloured picture would not consist of pure colours undiluted with white, the addition of further white will make the proportion greater than would occur in printing.

On the other hand, the eye is a very good judge of *hue*, a very small variation in the proportion of the colours (other than white being easily detected).

Much may be done, by training, to educate the eye to appreciate the relative luminosity of the colour, or, as it is usually termed, the "value" of the colour, and many artists are able to recognise variations in this almost as easily as in the hue itself. But the average person is not a good judge of this, and it is of much less importance than the hue. For instance, many people fail to see any improvement in a photograph of a landscape or a bunch of flowers taken through an orthochromatic screen, over a photograph of the same subject taken in the usual way.

But circumstances combine to make it difficult to recognise the addition of white even in a picture: the irregularities in the varnishing and the dust upon it are bound to add a proportion of white to the light reflected, and indeed it is very seldom that a picture is so hung that the varnish does not reflect a large amount of white, which we often do not notice until we attempt to take a photograph of it, and then very special lighting is found absolutely necessary.

5. *Printing with Black.*

As we shall obtain all our colours diluted with white, one is tempted to ask whether printing with a black ink will not enable us to get them pure; that is, as we can copy any colour, except for the addition of white, by the three printings in yellow, blue, and pink inks, could we not, by the addition of black, match the colours exactly?

Suppose it is the red we are matching. We shall get a red with a certain percentage of white. For instance, for 100 parts of red light reflected there may be 10 green and 10 violet. If we now print 10 per cent. black what will be the result? If the black is a good one, *i.e.*, if it absorbs uniformly all along the spectrum, we shall now have reflected 90 parts red, 9 green, and 9 violet. In other words, we merely darken the whole, without in the least altering the proportion of white. The addition of black will then not improve this purity of the colours; it will only make them "dirty."

6. *Production of Colour by successive Absorption of Light.*

There is a very fundamental difference between this and the production of colour by successive addition, as in Ives's triple projection. In the latter case, if any spectrum colour is thrown on the screen by one light—say, the green—and the same spectrum colour is transmitted also by a second light—red, for instance—the amount of that particular colour is doubled, and the result is an arithmetical addition of luminosity.

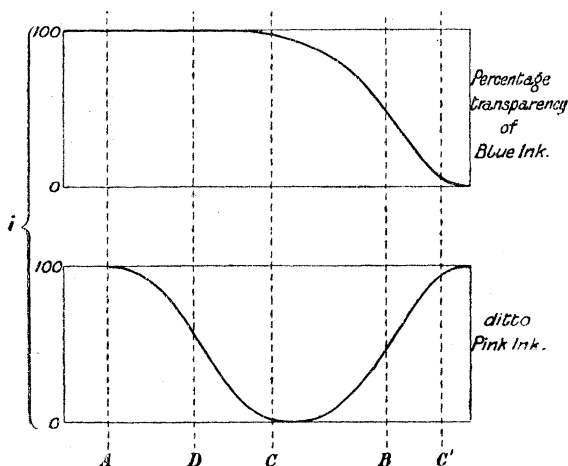
But in printing, if a spectrum colour is completely absorbed by one ink—say, the pink—and also absorbed by another—perhaps the blue—the total light of that colour absorbed by the superposition of the inks is not twice that removed by a single one. The absorptions do not successively subtract light. If two inks each would separately transmit one-tenth of the light of a given wave-length, the two inks together would transmit not one-twentieth, but only one-hundredth, of the light of that colour. The law is a geometrical and not an arithmetical one.

In "process" three-colour printing this is of the utmost importance, for there the inks are always printed full strength, and the tint is regulated by the size of the dots, that is, by the percentage area of the paper which is covered by the ink. The dots are produced by placing a ruled screen in front of the negative while it is being exposed, and the dots at any part, when developed and etched, have areas very nearly proportional to the intensity of the light which fell upon that part. Each colour is produced by similar dots, and when the three inks are printed these dots partly overlap one another. As the ruling is very fine and the dots are very closely spaced, it is impossible in

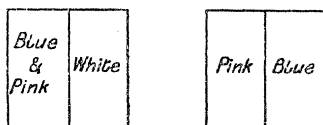
ordinary commercial printing to control the extent to which they overlap; that is to say, the successive impressions will, owing to minute differences in "registering" the paper, have the dots of the different colours more or less displaced in relation to each other.

If, for instance, the blue dots and the pink dots happened each to exactly cover half the area, in one impression they might be exactly superimposed, and in another they might hardly overlap at all. In the former case half the area will be printed with both pink and blue, and half will be white. In the latter case half the area is printed with blue and half with pink, and there is no white left. If these impressions are to be equally good, the resulting absorptions through the spectrum should be the same in the two cases.

Let the adjoining curves represent the percentage transparency of the inks. At A both inks are perfectly transparent, and as the colour is not absorbed by either, it can of course make no difference to this colour if the dots are superimposed or not.



At B each colour absorbs half the light and reflects half the light. When the dots are printed adjacent to one another, half the total light of that colour will be reflected. When they coincide, half the paper is



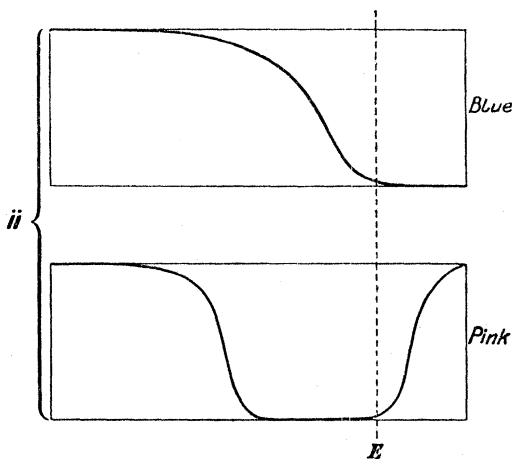
left white: this reflects all the light it receives, which is half the total. The blue ink on the other half absorbs half the light which it

receives of this colour (B), and the pink absorbs half of what is left, so that finally a quarter of the light falling on that half of the paper—or one-eighth of the total light of this colour—is reflected. Adding this to the white reflected from the white half, we see that *five-eighths* of the total light of this colour is now reflected, instead of the *one-half* which is reflected when they are adjacent.

The colour at C is entirely absorbed by one ink and is unaffected by the other. The pink ink absorbs the light which falls on its half, and as the blue ink does not affect this colour it will not matter whether it is on the pink or adjacent to it.

Also at D, where one ink is partly transparent and the other completely so, it cannot matter whether the perfectly transparent one is above or adjacent to the other.

If one or other of the inks is perfectly transparent at every point, the extent to which the dots overlap is immaterial. If the absorptions of the inks



overlap one another, as in Diagram 2 at E, the results are much worse. This colour is entirely absorbed by each ink. If then the dots are adjacent, the colour is absorbed everywhere and none reflected. But when they coincide half the paper is white, and therefore half the total light of that colour is reflected. Thus the reflected light of that part of the spectrum where the absorptions overlap will vary from nothing up to half the total light.

In the case of the pink and blue inks this overlapping would occur at the yellow, where their absorption terminates, the very brightest part of the spectrum. According to Abney's curves, the luminosity of the yellow from $\lambda 56$ to $\lambda 60$ is about 50 per cent. of the whole white light.

Thus the addition of one-half of this band of light would be sufficient

to entirely spoil the result, and this addition would be produced by a displacement of the dots of about $1/3000$ th of an inch.

It is obvious, then, that in process work the absorptions of the inks *must not overlap*. On the other hand, if we are to be able to produce a black, the absorptions *must meet*. Thus in process work (in the case of the blue and pink inks at least) the absorptions must just meet without overlapping, and therefore they *must end abruptly*. It will not matter so much in the case of the yellow and pink inks, for the part of the spectrum where their absorptions meet is of far inferior luminosity.

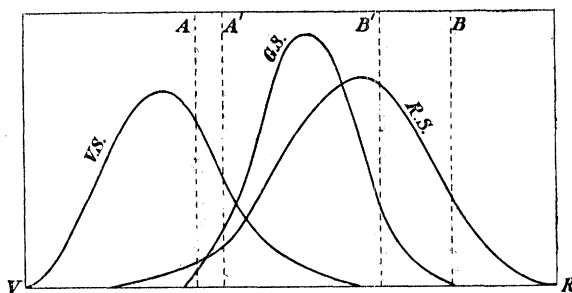
In collotype printing this difficulty would not arise in the same way. Here the tint is produced by the strength of the colour, the lighter shades being produced by covering the whole surface with a thin layer of ink. In this case there would be no uncertainty due to imperfect registering. But at the same time the ink curves for a thick layer of ink are very different to those for a thin layer, as is shown by the measurements I have added of the curves of different depths of the same ink. Thus the proportions in which the three inks should be combined to match a certain spectrum colour will vary with the luminosity of that colour.

To illustrate this. Suppose an object all one colour, such as a cast, to be illuminated by a monochromatic light such as a sodium flame, and a photograph taken, the result ought to be a picture in shades of yellow. But if the curves of the same ink when printed to different depths are different so that the proportions of the colour sensations that it reflects are different, the proportion in which it is combined with the other two inks to match a bright yellow will not be the same as that required to match a dull yellow of the same wave-length. Thus if the filters are adjusted to give the exposures on the three plates that will reproduce that hue in, say, the high lights, it will not be correct in the half-tones, and will be still worse in the shadows. The ideal ink for this process would be one which would absorb the colours in the same relative proportions, whatever the depth to which it was printed. The curve would be one of perfect transparency up to the absorption band, then a sudden drop (for the full colour), say, to about 2 per cent., to remain at this height till the end. The curve should have no rounded corners.

It will be seen that the sharpness of the drop depends on the depth to which the colour is printed; thus for "process" work, where it is always printed to the same strength, the absorption can be made abrupt by using a very full tint. But if this is to be done, the ink must be a very transparent one to the part of the spectrum it is not supposed to absorb, for, as we have seen, it is very important that the same part of the spectrum should not be absorbed by more than one colour.

7. *Best Limits for the Ink Absorptions.*

Assuming now that the absorptions should be abrupt, the next question is, what limits should they have to give the best effects? If one is at A and the other at B in the accompanying diagram—in which equal heights of each colour would give white—the yellow and



blue inks will be complementary to the colour sensations, but they will be very pale, for they reflect large quantities of all three colours. The pink ink will be very dark, it will reflect only a very small amount of the red sensation, and as the violet reflected is not equal to the red, it will not be complementary to the green sensation. It could be made so by shifting A further to the violet, so as to reduce the violet reflected by the pink ink—in fact, until the areas of the red and violet are equal. It is obvious that this would give absurd inks; the yellow and blue would be so pale as to be mere tints, and the pink so dark as to appear purple.

If the absorptions are supposed to end at A' and B', the spectrum will be much more evenly divided between the three colours, each will be slightly pale, but now *neither* will be the complementary of the colour sensation. For instance, if the yellow ink is to be complementary to the violet sensation, it should absorb either violet or violet and equal amounts of red and green, measured by equal areas on the above curves. But at A' (or at any other line between A and B) the green sensation is evidently in excess. In the same way if the absorption of the blue ink extends from the red end of the spectrum beyond B, green is absorbed as well as red, and the ink cannot be complementary to the red sensation. To make this ink complementary, it must have another absorption band somewhere in the violet, and the difficulties we have been considering above will be introduced, for there will then be a part of the spectrum absorbed by more than one ink, namely, by the yellow and the blue, and in places where these inks are printed side by side the absorption of this colour will be twice as great as in those where they are on top of one another.

Thus the blue ink should *not be the complementary* of the red sensation. And in fact the ink will be far from complementary to the red. I find by taking the areas of Abney's curves up to B' with a planimeter, that it should very nearly match the green at about λ 50.3.

The yellow ink would be made complementary to the violet sensation by ending the absorption at A, but it would be very pale, for it would transmit a large amount of white, as there is a large proportion of the violet beyond A. The result will be an unnecessary amount of white in the final picture. The result will be better if the absorption reaches to A'. Thus this ink also *should not be complementary* to the violet sensation. The pink ink at the red end of the spectrum transmits red and green, and at the other end chiefly violet and green. If the violet is equal to the red it will be complementary to the green sensation. This can be achieved by moving A towards the violet end, but only at the expense of the yellow, which would be rendered very pale. So this ink also should *not be complementary*.

I have gone into this rather fully as it has so frequently been stated that the inks ought to be complementary to the colour sensations. In a very rough and general way this is true, but there is no advantage in such inks; on the contrary, they would, even if obtainable with abrupt absorptions, not be so good as others that are not complementary. The only possible advantage in using complementary inks would arise in the case in which the absorptions were abrupt, for then the filters would be complementary to Maxwell's curves. But the advantage here is only one of theory, the printed result being, as we have seen on the preceding page, a picture with a very large excess of white. As in practice the filters would be adjusted by trial, even this advantage is illusory.

To summarise. The inks could only be complementary—

1. If they had abrupt absorptions with limits near the ends of the spectrum, when the colours would be in some cases mere tints; or
2. If they have overlapping absorptions, or if some colours are absorbed by more than one ink, both of which are bad, since the resultant absorption will follow a geometrical and not an arithmetical law.

The exact positions of the limits will, no doubt, finally depend on the fact that there will be very few pigments which sufficiently fulfil the conditions, and the blue and pink ones with abrupt absorptions near the yellow will not be numerous. Still it is theoretically interesting to determine the best positions, apart from the difficulty of finding inks to suit. This will be best done by calculating the curves for different absorption limits and estimating the amount and distribu-

tion of the added white, and the luminosity of the resultant colours in each case. I have worked out the curves for two interesting positions of the absorption limits.

8. *Choice of Inks.*

The mere appearance of an ink, even when a rubbing on paper is taken, is not much guide, as the eye is not suited for selecting the inks by their hue; for they are double-colour inks, and we know that the eye cannot at all judge of the composition of a yellow or a blue. Yellow, for instance, might be spectrum yellow, or a mixture of any part of the red end of the spectrum with almost any part of the green. In other words, the eye cannot distinguish between yellows of a great variety of compositions. But the eye is a fairly good judge of the three primary colours—at least, of their hue—so that if the inks be printed in pairs a fair estimate can be made. The violet produced by the pink and blue inks should be almost pure, containing only a small percentage of white, as the red and green are almost equal. The green should very closely match the green sensation. But the red will contain some green, and should about match the hue of λ 67·4.

But the only true test is the spectroscopic one. Examined spectroscopically—

1. The inks should each absorb one band of the spectrum in the red, green, and violet respectively.
2. Must transmit the remainder unaffected.
3. Must absorb the whole between them.
4. For half-tone work the absorptions must end abruptly, especially in the case of the pink and blue inks.
5. The limits of the absorptions should be so chosen that the added white may be distributed through the spectrum as nearly as possible proportionally to the luminosity of the spectrum, and that
6. The luminosity of the resulting colours in matching the spectrum should be as high as possible.
7. The inks will not be complementary to the colour sensations.

For (3) the absorptions must meet, for (2) they must not overlap; thus (2) and (3) lead to (4).

The above conditions are, I believe, sufficient. I have examined a large number of inks supplied by different firms to see how far they are realised by those at present in use.

As the orange, yellow, green, and blue-green parts of the spectrum are far brighter than the rest, it is here that any conditions laid down must be specially observed. In the blue and violet and in the deeper red it is not so important.

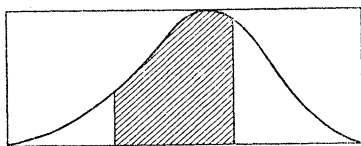
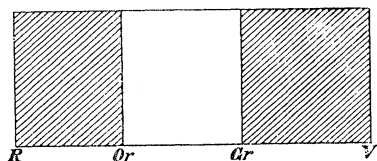
Thus, if the pink ink is only partly transparent in the blue and has not an abrupt absorption there it will not much matter, provided it is perfectly transparent in the red and orange, and has an absorption commencing suddenly in the yellow. Such an ink is Fleming's "Theoretical Red," No. 1303.

The yellow must be perfectly transparent in the red, orange, yellow, and green with an absorption commencing in the blue ; but it does not matter if it is not very sudden.

The blue ink must be perfectly transparent in the green, blue-green, and blue ; there must be an abrupt absorption in the yellow. The violet does not matter so much. So far I have found no ink to fulfil these conditions. All the ordinary inks are too opaque in the green and blue-green.

9. *The Luminosity as well as the Hue of the Spectrum must be Matched everywhere.*

It must not be forgotten that most natural colours are composed of a large range of spectrum colour, and the resultant hue will depend on the proportion in which these colours are compounded. Now although the eye is a bad judge of the luminosity of the colour, it is a very good judge of the resultant hue of a compound colour (and thus indirectly it is of course able to compare the relative luminosity of its components as far as this luminosity is due to colour as distinct from white). It follows that the component colours must be rendered in their correct proportions. When a negative is exposed through its light-filter the light that will reach it from a natural object will be the sum of all the colours that the object reflects, each rendered according to the curve of the filter. Suppose for instance that all the light from the orange to the green is reflected by the object as in the first figure.



Then if the second figure represents the curve of the light-filter, the area of the shaded portion gives the amount of transmitted light

and (supposing for the moment the plate to be equally sensitive to all colours) will be proportional to the final opacity of the plate. If the filter curves have been arranged so as to match the spectrum in luminosity as well as hue, then the resulting print will reflect the sum of light which is equivalent in ray composition to the sum of this band of spectrum colour, and will therefore match it in dominant hue. But if the spectrum has been matched in hue only, the resultant colour will be the sum of the colours in this band mixed in the wrong proportions, and will not have the same dominant hue as the original.

10. *The Light-filters.*

If the inks are neither perfectly transparent or perfectly opaque to each colour, so that there are no parts of the spectrum to which they are only partially transparent, and if also the absorptions do not overlap, the resultant colour will be that left after successive subtraction by the three inks, and will be the same as would result from the successive addition of the three complementary lights in triple projection; and then the filters can be found by determining the dominant hue of the complementary colour.†

The curves for the filters can also be calculated in this case by finding what percentage area the inks should cover so as to transmit light whose composition, in sums of the colour sensations, is everywhere that given by Maxwell's curves. This is how I calculated the curves for the hypothetical inks.

But when we deal with practical inks these conditions are far from being fulfilled, and the filters cannot be found by dealing with the complementary lights. For, as we have seen, since the same colour is absorbed, at least to some extent, by more than one ink, the arithmetical law of absorption will not hold, and the colour resulting from the printing with the inks is not that produced by the combination of the complementary lights. So that in practice we shall have to adopt some other method, *which must be experimental, and must be founded upon impressions of the actual inks.*

11. *Depth of Colour and Process Printing.*

One of the greatest practical difficulties in printing with only three colours, and especially with such strong colours as red and blue, is to ensure that the quantity of ink shall not vary from impression to impression. Usually, very slight variation in the amount of ink is accompanied by a change in the whole tone of the picture. This is because with most inks the curve of absorption is different for different depths of colour, and thus the proportion of the three fundamental

† Abney, 'Photographic Journal,' January, 1900, p. 121.

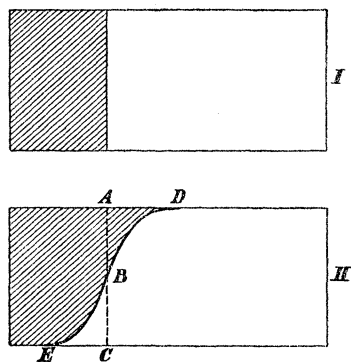
colours left in the reflected light also will vary with the depth. This will not matter much when a large number of colours—twelve or fourteen—are used, as the average amount of red and blue put on by the successive inks will be nearly constant; but with only one ink of each colour the effect is very marked.

This effect is greatest when the curve for the ink is a gradual one, for as the quantity of ink is increased in an arithmetical progression, the absorption of any given colour will increase in a geometrical progression, assuming, of course, that the ink is transparent. Thus, unless the distributing apparatus of the machine is very perfect indeed, it will be impossible with such an ink to obtain uniform impressions.

Were the ink one with abrupt absorption, that is, one which was very transparent up to a certain colour, and then nearly opaque, slight variations in the quantity of ink would have very little effect, and one of the greatest difficulties would at once disappear.

12. *Effect on the Purity of the Colour of an Abrupt Absorption.*

It might perhaps be supposed that inks which reflected most light in the parts of the spectrum for which they are to be transparent, and gradually reflect less up to the part they are to absorb, would be nearer approximations to monochromatic inks than those whose absorptions begin abruptly. That this is not the case can be easily seen if we compare the two red-absorption inks opposite. In the second we have replaced the reflected light ABD by BEC. Now Maxwell's curves show that the proportion of red to green in BEC is much greater than it is in ABD, so that the absorption of the red is not so complete in II as



it is in I, while the transparency to green is less. Thus II would not be so good an ink as I. When, for instance, this and the violet absorption ink are printed, which ought to leave green only, there would be some red left, and this in greater amount than with the ink I. This would

mean that to match green more violet would have to be left to form white with the excess red, and the dilution of the green with white would be greater.

13. Summary of Reasons for using Inks with Abrupt Absorption.

- i. The colour will not be affected by slight variations of register.
- ii. Variations of the amount of ink in the several impressions will not be so important.
- iii. The colours will be much purer, being mixed with less white.
- iv. The curves for the filters can be found if desired by determining the complementary light, and working as Abney has described in his paper in the 'Photographic Journal' already mentioned.

This last will, however, only be possible if the curves are nearly 100 per cent. up to the absorption in each case, and so far I have found no set of inks, nor even a single ink for which this is the case.

PART II.

14. Calculation of the Curves for Theoretical Inks.

By theoretical inks I merely mean inks which are perfectly transparent until the absorption commences, and then are perfectly opaque. With such inks we can calculate the percentage area to be covered to match any given colour by considering the absorptions they produce.

I drew on a large scale the curves found by Abney for the ray composition of the normal spectrum, in which equal ordinates form white. I drew vertical lines where the ink absorptions should terminate, and with a planimeter I found the areas of the curves up to these lines. Thus I obtained the ray composition of the inks.

Thus, taking the vertical lines at wave-lengths 49 and 59, I found the following areas. From the violet end of the spectrum up to λ 49—

Violet.....	122·4	or	Violet.....	119·4
Green.....	4·8		Green.....	1·8
Red	3·0		Red	0·0
			White	3·0

This will be the part of the spectrum absorbed by the yellow ink. From λ 49 to λ 59 the areas were—

Violet.....	25·8	or	Violet.....	0·0
Green.....	128·0		Green.....	102·2
Red	79·5		Red	53·7
			White	25·8

This will be the part of the spectrum absorbed by the pink ink. From λ 59 to the end of the spectrum they were—

Violet, 0. Green, 15·6. Red, 65·8.

These add up to—

Violet, 148·2. Green, 148·4. Red, 148·3.

Thus the ray compositions of the inks are—

	Yellow ink.	Blue ink.	Pink ink.
Violet	25·8	148·2	122·4
Green	143·6	132·8	20·4
Red	145·3	82·5	68·8

The blue ink matches λ 50·4 and white, and the yellow will match λ 58 with white.

I now took the ray composition of some spectrum colour, and found by successive approximation the area to be covered by the inks to match it. Thus λ 44 has a ray composition of—

Red.....	1·47	equivalent to	Red.....	1·27
Green.....	0·20	„	Green.....	0·0
Violet.....	76·68	„	Violet.....	76·48
			White.....	0·2

Then if 66 per cent. of the area is free from yellow ink, 66 per cent. of the light the yellow would absorb will be transmitted, namely—

Red, 0. Green, 1·2. Violet, 76·6. White, 2.

So if 5 per cent. of the area is left free from blue ink it will reflect in addition—

Red, 3·2. Green, 0·78.

The pink ink must be printed all over. Then we shall have left—

Red.....	3·2	equivalent to	Red.....	1·22
Green.....	1·98	„	Green.....	0·0
Violet.....	78·6	„	Violet.....	76·46
White.....	2·0	„	White.....	3·98

which matches the spectrum colour except for an excess of 3·78 white.

In the same way I worked out the percentage areas to be left free from colour to match each of the wave-lengths 38, 40, 42, &c., up to 70, and have found in each case the excess of white.

I selected the above positions partly because they divide the spectrum about evenly, but also because most of the pink inks have their absorptions at about λ 59, and the yellows at about λ 49. As the blue inks are all bad, it is no use considering where their absorptions are.

Another interesting place for the absorptions to end is at the points of intersection of the ray composition curves, namely, at λ 50.5, where the green and violet curves cross, and at λ 57.5, where the green and red cross. It is useful to compare the excess of white and its distribution in the two cases.

The results of these calculations are given in the following tables:—

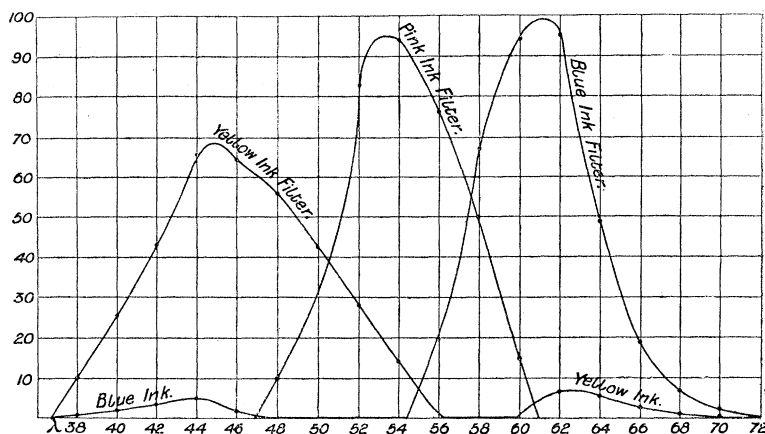
Absorptions at λ 49 and λ 59.

Wave-length.	Luminosity.	Percentage area not covered by inks.			Excess of white.
		Yellow.	Pink.	Blue.	
38	0.25	10.4	—	0.8	0.63
40	0.75	25.5	—	2.0	1.5
42	1.3	43.4	—	3.18	2.6
44	2.1	66.0	—	5.0	3.8
46	3.5	64.0	—	1.4	0.9
48	7.5	56.0	10.0	—	6.7
50	18.0	42.4	31.4	—	19.2
52	56.0	28.0	83.2	—	23.4
54	80.0	14.3	94.4	—	35.5
56	95.0	—	76.0	19.7	16.7
58	99.0	—	50.0	66.5	12.2
60	85.0	—	16.0	94.5	4.1
62	59.0	6.2	—	95.5	0.2
64	26.0	5.3	—	48.4	0.2
66	10.0	2.35	—	19.3	0.07
68	3.2	0.92	—	6.4	0.03
70	0.9	0.3	—	1.8	0.01
					125.74

The sum of the ray compositions for the same wave-lengths is 1120. Thus the excess of white is about 9 per cent.

It will be seen that the excess is chiefly in the brighter part of the spectrum, though it seems rather in the green than the yellow.

These calculations entirely agree with the arguments advanced in the earlier part of this paper to show that the light filters for three-colour printing should be entirely different from those used for three-colour projection.



No. 27.—Filter curves for inks with abrupt absorptions at λ 49 and λ 59. The ordinates show the percentage area to be left free from ink, and therefore indicate the opacity to be obtained on the corresponding negative, or—for an orthochromatic plate—the transparency of the filter.

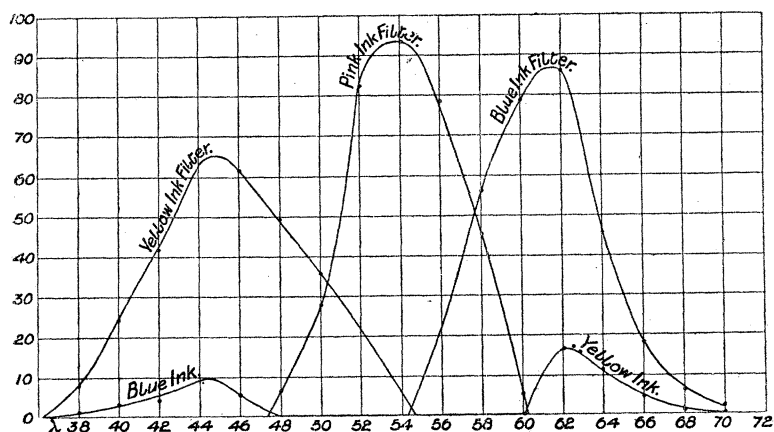
Absorptions at 57·5 and 50·5.

Wave-length.	Percentage area left uncovered by ink.			Excess of white.
	Yellow.	Pink.	Blue.	
38	8·5	—	1·1	1·1
40	24·6	—	3·3	3·6
42	42·2	—	5·9	6·4
44	63·7	—	9·0	9·6
46	61·6	—	5·5	6·0
48	49·3	6·3	—	2·2
50	35·7	28·0	—	6·8
52	19·9	82·8	—	23·4
54	5·3	93·8	—	10·7
56	—	78·5	21·7	7·0
58	—	45·8	57·0	5·2
60	—	5·1	79·0	0·6
62	16·5	—	86·0	22·2
64	11·2	—	46·0	15·2
66	4·2	—	18·0	5·9
68	0·6	—	5·9	1·9
70	0·2	—	2·0	0·6
				128·4

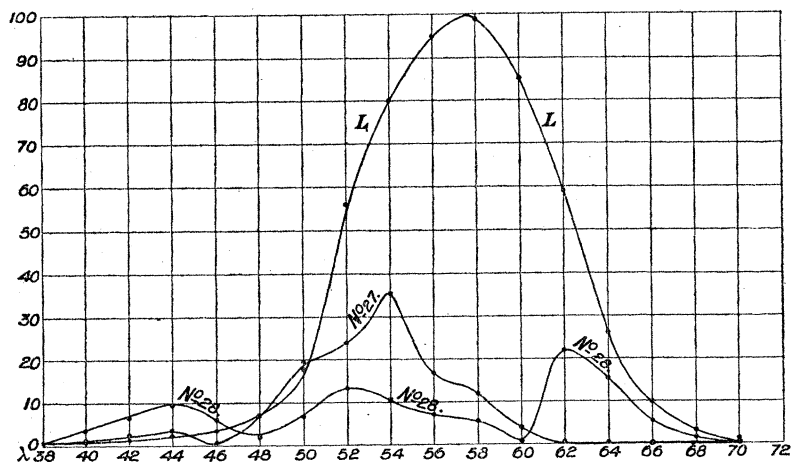
The total excess of white is about the same, but the distribution is not so good, as the ends of the spectrum, when the luminosity is small, have large excesses.

The absorptions of the above inks are—

Yellow.		Pink.		Blue.	
Violet.....	135·6	Violet.....	12·8	Violet.....	0·2
Green.....	10·6	Green... ..	105·2	Green.....	33·0
Red	5·1	Red	55·9	Red	87·3



No. 28.—Filter curves for inks with abrupt absorptions at λ 50·5 and λ 57·5.



No. 29.—Curves showing the excess of white which the filters will give in the finished print of the spectrum with the theoretical inks. LL, curve showing luminosity of the spectrum.

The results of the calculations are exhibited in the accompanying diagram, curve No. 28, and the distribution of the excess white which they involve is shown in the diagram, No. 29, together with Abney's

luminosity curve of the normal spectrum. It is obvious from them that inks whose absorptions terminate at λ 49 and λ 59 will be better than those terminating at the points of intersection of the ray composition curves.

PART III.—EXPERIMENTAL.

15. *The Apparatus.*

Throughout the following experiments we used a copy of Abney's colour-patch apparatus. The only difference in the main instrument was the use of a right-angle prism in front of the collimator slit to reflect the light from the lantern down the tube. This enabled us to set the apparatus up much more nearly in a direct line, and therefore to arrange it all on an ordinary long table.

The collimator was about 13 inches long, with a $2\frac{1}{2}$ -inch achromatic lens. The slit was one of Elliott Brothers'.

The prisms were equilateral prisms cut from a very fine specimen of white glass especially selected for this purpose. The face of each was about $2\frac{3}{4}$ by $2\frac{1}{4}$ inches.

The projecting lens was an achromatic one, $2\frac{1}{2}$ inches diameter and about 30 inches focus. It produced a visible spectrum 8 cms. long, which was very bright and sharp.

The slit was mounted in a brass frame arranged to slide horizontally across the spectrum in its focal plane. It was about $2\frac{1}{2}$ inches high, and usually about .5 mm. open. A small 1-inch right-angle prism, carried by the same frame, was arranged to reflect the light from the top half of the slit to one side on to a second and larger prism, also carried on this frame, which reflected it forward.

Combining Lenses.—These were two 6-inch lenses of about 20 inches focus, one of which received the light from the lower half of the slit and focussed it on the patch. This is the direct beam. The other lens focussed the reflected beam on the same patch. This patch is the image of the aperture of the double-image prism mentioned below. The distances of the lenses were adjusted until the horizontal edges of the patch were sharp, and were then rotated slightly round a vertical axis until the patch remained stationary as the slit was moved across the spectrum.

The patches were formed by cutting a small square window in a piece of black velvet pasted on a card. To avoid stray light the card was placed at the far end of a box, 2 feet long, blacked all over. The mouth of the box was covered with brown paper, and holes were cut in this just large enough to admit the two beams. A part of one side was removed to enable the patch to be observed. A blackened pillar was placed, as in the Rumford photometer, to obtain a sharp dividing line, one-half of the patch being lighted by

the direct beam and the other half by the reflected beam. The two patches were adjusted (before the pillar was inserted) to exactly coincide, so that any want of uniformity in the illumination might affect them equally; this we found most important. The patches were slightly larger than the window, so that it was lighted up to its edges. The pigment to be compared was always placed in the reflected beam, the white beam falling on a piece of the same paper that the pigment had been printed upon.

The sector was one made by Hilger upon the lines laid down by Abney. It closes entirely, and opens to one-half of the full beam. The aperture is divided into 100 parts, so that percentage aperture can be read directly. It has to run at a high speed to obtain good results. I found that there was a lot of backlash in the sleeve, amounting to about two divisions. To avoid the error due to this, it must be stopped always in the same way. I found by exactly closing the sector while it is running—which can be done very accurately by looking through it to the light and adjusting it until the light is entirely extinguished—and then stopping it *by the sleeve*, the reading is nearly always zero. If in addition the handle has last been moved in the direction to *close* the aperture, the reading was always found to be correct. These experiments took a long time. The sector was placed in the direct beam just in front of the box spoken of above.

As the direct beam is cut down to one-half by the sector, even when the latter is fully open, it is necessary to reduce the reflected beam also to enable a balance to be obtained. This was done by attaching a Nicol's prism to the frame carrying the slit and the two right-angled prisms in the beam between the two latter. As the light has already been polarised by the double-image prism, it is possible by rotating this Nicol to reduce its luminosity to any desired extent.

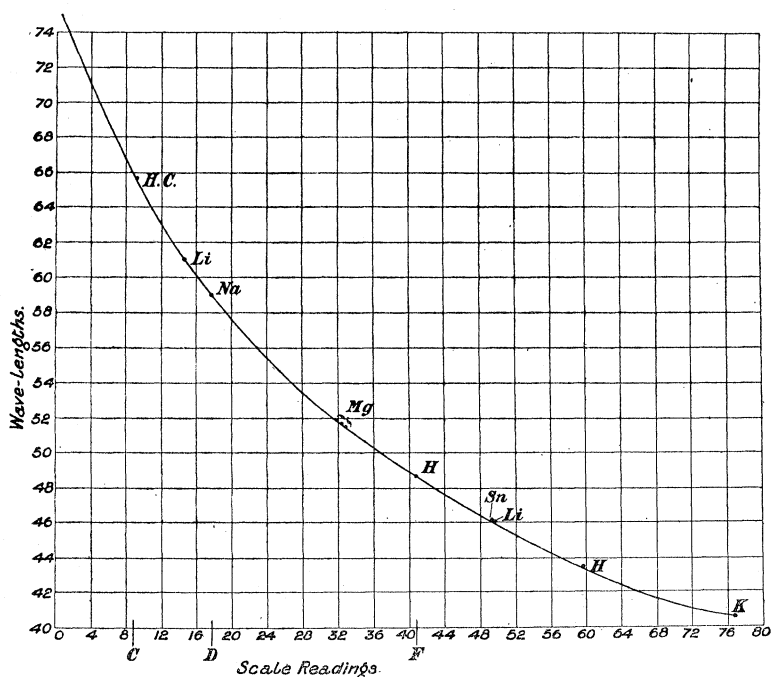
The double-image prism was about 1 inch square aperture. It was at first mounted against the collimator lens, but finally was placed against the projecting lens. Its adjustment is of very great importance, and it was found impossible to set it with sufficient accuracy by hand. It was therefore mounted in a brass frame which could be given a very slow rotation by means of a screw, and after the prism had been adjusted as nearly as possible by hand, it was finally corrected by this screw. Its rotation causes the two spectra, into which it divides the original one, to move relatively across one another, and therefore varies the colour which passes through the slit. The prism is rotated until, with the slit in the yellow, the two beams are exactly the same colour. As the slightest alteration here is very easily perceived, the adjustment can be effected very accurately, and will then be correct throughout the spectrum.

A glass Zeiss millimetre scale was temporarily attached to the top of the camera, and a pointer on the frame carrying the slit moved

along the scale and indicated the position. The D line was at 50 on this scale. The temporary scale was afterwards replaced by a permanent one on which the D line was at 17·7, and to avoid confusion, the early scale readings have been reduced to this scale by subtracting 32·3 from each.

16. *Scaling.*

For this, the slits were removed and the spectrum was received on a strip of celluloid placed between a glass plate and a glass Zeiss millimetre scale. Various substances were then placed on the arc, such as Li, K, Na, Mg, and the bright lines read. A H-tube was also placed at the collimator slit. The results are plotted in Curve No. 1 :—



No. 1.—Curve connecting the scale readings with the corresponding wave-lengths.

Line.	Scale.		Wave-length.
	Old.	New.	
D.....	50·0	17·7	5889
Li, red.....	39·3	7·0	6708
Li, orange	46·8	14·5	6102
Li, blue	81·8	49·5	4603
K, red.....	30·5	1·8	7680
K, violet.....	109·0	76·7	4045
H, C.....	41·2	8·9	6563
H, F.....	73·0	40·7	4860
H.....	92·8	59·5	4340
Mg, green	64·1	31·8	5183
"	64·4	32·1	5172
"	64·5	32·2	5167
Sr, blue	81·6	49·3	4607
Ba.....	56·5	24·2	5535
Ba.....	57·5	25·2	5492
Ca.....	37·1	4·8	6933
Ca.....	56·3	24·0	5543
Ca.....	98·3	66·3	4226

The D line on the new scale was at 17·7, and as it was also a millimetre scale, the new scale readings will all be 32·3 less than the previous ones.

To test this the following readings were taken :—

Stem.	D.	Li, red.	Li, blue.	Mg.
Old scale.....	50·0	39·3	81·8	64·5
New scale	17·7	7·8*	49·5	32·3
Difference.....	32·3	31·4	32·3	32·2

17. Determination of the Curves of Practical Inks.

January 1, 1900.—We set up the apparatus with the two right-angle prisms and obtained the patches. We found some difficulty due to stray light, which we nearly eliminated, so far as the outside of the apparatus was concerned, by inclosing it with screens. A cloth was placed over the arc and over the collimator and prisms, and a blackened canvas screen was hung up to divide the end of the room where the observations were made from the end containing the light. But when matching pigments in a part of the spectrum in which they were nearly black, we were still able to perceive a small difference of colour in the two patches, which was due to internal reflections on the

* Probably an error in reading.

collimator and camera bodies and at the surface of the double-image prism. We reduced this as far as possible by the liberal use of stops, but could not entirely avoid it. This is not wonderful when we consider the small luminosity of the ends of the spectrum, and remember that we were often only using 2 or 3 per cent. of this feeble light. It is obvious that a very small amount of reflected light would be easily noticed. On some occasions the patches were observed through coloured gelatines to help to cut this off.

For these adjustments I am greatly indebted to my friend, Mr. Alex. A. Tallent, who worked most enthusiastically in getting the apparatus together, and who has assisted me throughout in the experiments.

The relative intensity of the direct and reflected beams was not constant through the spectrum, and it was necessary to measure it. For as the light strikes the second refracting prism at an angle very near the polarising angle, it is thus partly polarised when it reaches the double-image prism, and therefore the two beams into which this prism splits it will not be of equal intensity, and this inequality is more marked for some colours than for others.

March 27.—I measured a yellow and red ink by Messrs. Mander, of which I had made rubbings. Probably due to small deposits of moisture which form on the prisms and lens—owing apparently to the glass being more than usually hygroscopic—the inequality above mentioned varies from day to day. The moisture probably reduces the polarisation by reflection at the surface of the prism, and thus alters the relative luminosity of the beams. It is not advisable to clean the surfaces very often for fear of spoiling the adjustments. It is therefore necessary to frequently re-determine the relative brightness of the patches.*

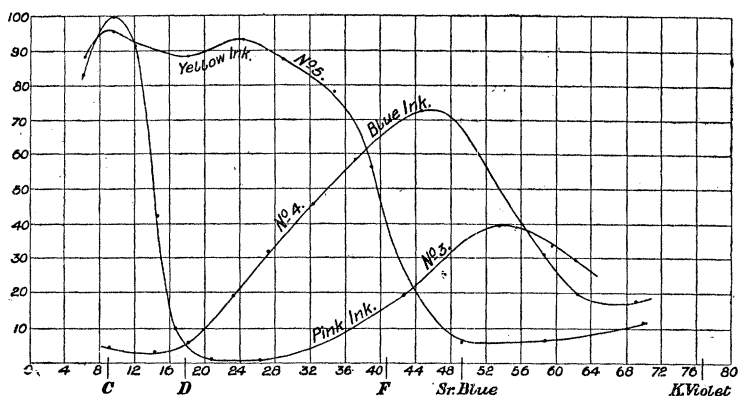
We found a great improvement in the accuracy of our readings as we became more experienced, and also as the stray light was more perfectly eliminated. The chief difficulty remaining was caused by the unequal density of the inks over the patch, which is far more obvious when measuring them in a monochromatic light than it is when casually observed in ordinary light.

18. *Description of the Curves.*

With this apparatus I measured the proportions of the light of each colour throughout the spectrum that was reflected by several of the inks sold commercially for three-colour printing.

I measured such a set of inks sent by Messrs. Fleming and Co., and described as "Theoretical Inks" for three-colour work. They are not permanent, but that would not be important for book illustration,

* The apparatus was entirely re-adjusted after No. 2 was measured.



Curves from Fleming's "Theoretical" Inks.

though it would be fatal for advertisements or pictures to be exhibited.

Jan. 2.—The Pink Ink No. 1303 (Curve No. 3) seems very satisfactory, as its absorption begins very rapidly in the orange, whilst the curve rises again fairly high in the blue.

The Blue Ink No. 1265 (Curve No. 4) is far too opaque in the yellow and green. The absorption commences very gradually, and there is also an absorption at the violet end. The ink thus leaves much to be desired. It should rise at about 18 (scale) to 100 per cent., and stay at that to the end, and this it is very far from doing.

The Yellow Ink No. 1271 (Curve No. 5) is very transparent well into the blue, and will probably do well.

Messrs. Fleming also sent me a set of rubbings of the inks they ordinarily supply for photochrome work. These inks are permanent or nearly so :—

Photochrome Red No. 1197 (Curve No. 6) has an abrupt absorption rather further in the yellow than the "Theoretical Red," and is so far better, but is far less transparent in the blue, rising at the blue lithium line only to about 5 per cent. as against the 35 per cent. of the "Theoretical Red."

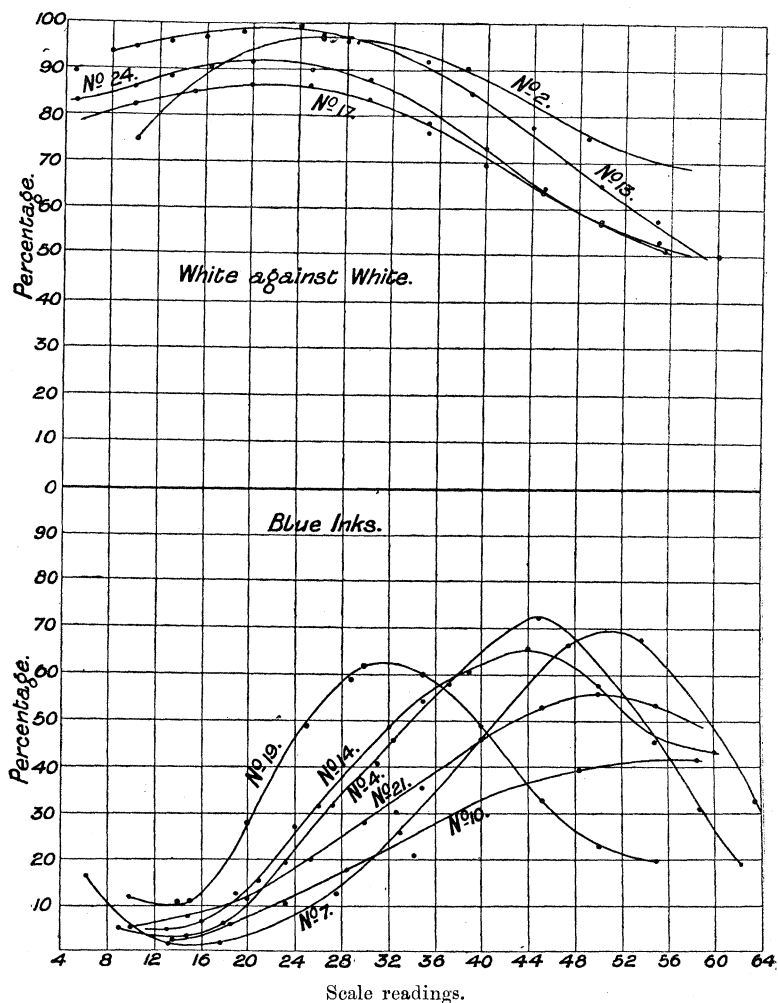
Photochrome Blue No. 1199 (Curve No. 7) rises fairly rapidly, but much too late, namely, in the blue-green instead of the yellow. It will not be possible to make bright greens with this ink.

Photochrome Yellow No. 1198 (Curve No. 8).—The absorption is more abrupt than that of the previous yellow, but it is earlier, that is to say, it is not sufficiently transparent to the blue. It ought to occur at about 46 or 50 instead of 32 (scale).

I measured a set of inks which were printed for me by Mr. Gamble, of the Bolt Court School of Photo-lithography. The pink ink (Curve

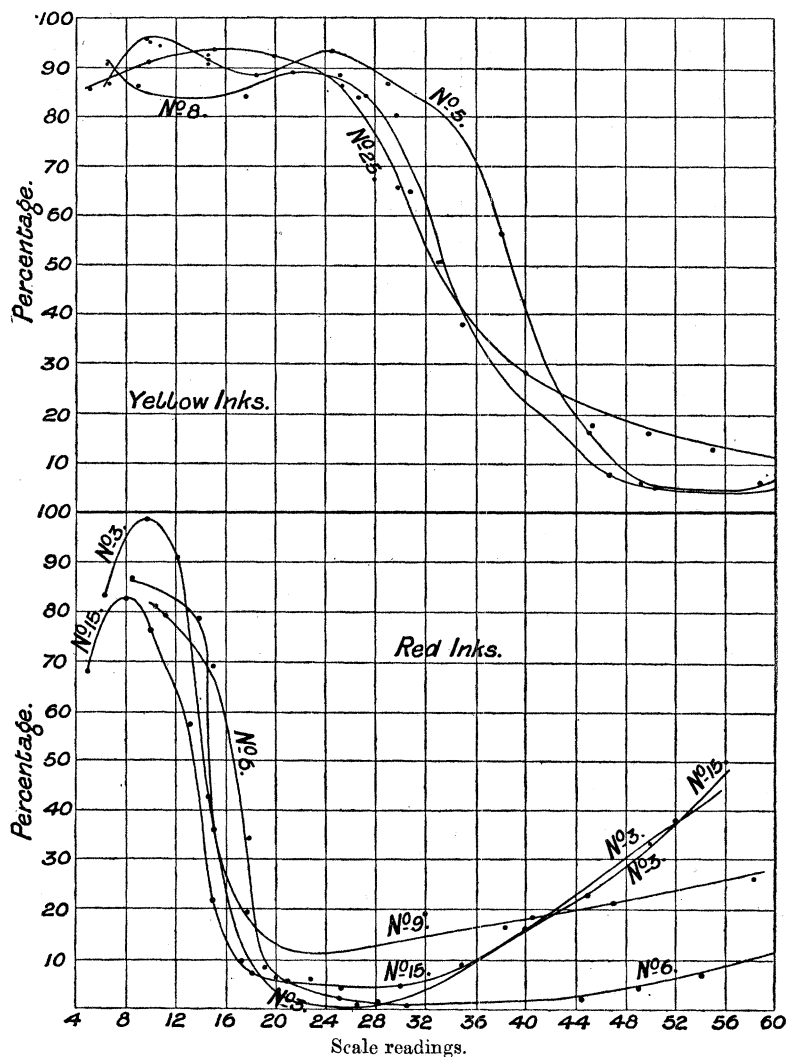
No. 9) had no very great absorption anywhere. The blue ink (Curve No. 10) was not transparent enough anywhere, especially in the green.

In another set of inks printed by Mr. Gamble the blue (Curve No. 14) had very much the character of the first one above, as had also the pink and yellow (Nos. 15 and 18).



The White against White Curves show the effect of the film of moisture on the prism, on different dates.

Mr. Gamble formed a *Red* by printing both the pink and yellow inks on the same paper, and it is interesting to compare the absorptions



of the red so obtained with the absorptions calculated from those of the pink and yellow inks. As it will at the same time show how the curves were obtained in the other cases, the following table is inserted :—

Scale.	Average aperture for yellow ink.	White.	Per cent.	Per cent. for pink ink.	Product of percentages.	Per cent. for red ink from experiment.
10	78.5	82.0	95.7	76.7	73.3	75.7
15	79.0	85.0	93.0	21.9	20.4	20.3
20	79.5	86.5	92.5	6.37	5.9	4.5
25	77.0	86.2	88.5	4.4	3.9	5.7
30	67.7	83.5	80.0	4.7	3.76	5.8
35	34.0	76.5	44.5	9.0	4.0	6.3
40	15.0	69.5	21.6	16.2	3.5	7.4
45	7.7	64.4	11.94	23.0	2.75	7.1
50	6.0	57.3	10.46	33.4	3.5	6.2
55	4.5	50.5	8.9	46.6	4.16	6.1

In the above table the column headed "Average aperture for yellow ink" gives, for each spectrum colour, the aperture of the sector which made the light reflected from the white patch equally bright with that reflected from the patch printed with the yellow ink. The next column, headed "White," gives the aperture of the sector which made the two patches equal when both were white. This column would have been constant but for the polarisation by reflection at the surface of the refracting prism already referred to. From these two columns the proportion of the light reflected by the ink as compared with that reflected by white paper can be calculated. The result appears in the column headed "Per cent."

In the same way the percentage light reflected by the pink and the red patches were found, and are entered in the fifth and seventh columns. In the sixth column I have calculated the light which should be reflected from a paper printed with both the yellow and the pink inks. The agreement is very fair. The actual red reflects more in all the darker parts than calculated. This may be due to the yellow not being quite so heavily printed, but it is more probably because the pink ink, which was uppermost, was not quite transparent, and some light was reflected directly from it instead of from the white paper beneath. Such light would not suffer by the absorption of the yellow ink.

The "red" resulting from these printings has a very orange hue, as was to be expected.

A yellow ink by Mander, No. 226 (Curve No. 25), was less heavily printed than those previously examined, and therefore seems better in the green and blue green, but the absorption in the violet suffers in consequence.

A pink ink by Mander, No. 0227 (Curve No. 26), comes up fairly in the violet, but is not opaque enough in the green.

As the worst ink was always the Blue, I attempted to find a colour which should be more transparent in the green. I obtained a bright green rubbing from Fleming, but I found it was green not because it was more transparent in the green and yellow-green, but because it was more opaque in the blue and violet. There seems to be no ink in the market at present that is sufficiently transparent in the yellow-green.

19. *Attempt to find the Effect of a Difference in the Depth of the Colour.*

The intensity of the reflected beam is given (for any one colour) by $I' = I a^e$, where e is the depth and I is the original intensity of the beam. For another depth of tint f

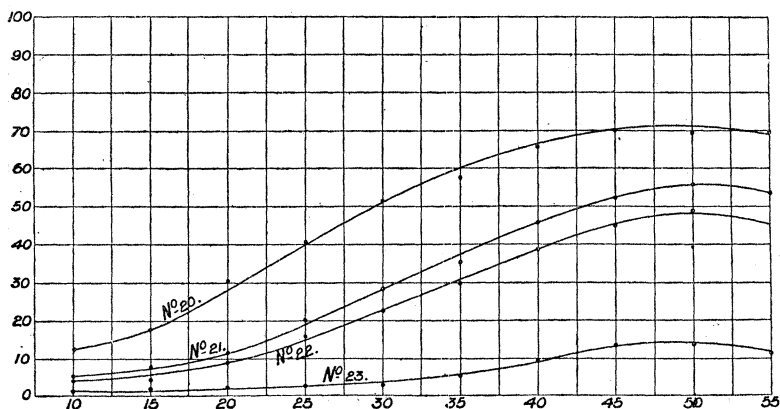
$$I'' = I a^f,$$

$$\log \frac{I'}{I} = e \log a, \quad \log \frac{I''}{I} = f \log a,$$

$$\therefore \frac{e}{f} = \log \frac{I'}{I} \div \log \frac{I''}{I}.$$

Thus the ratio of the log ought to be a constant through the spectrum. Neglecting the readings in which the aperture was too small to be taken accurately (owing to the backlash of the sector), and in which also the scattered light is of importance, the following results show a very close agreement with theory.

The inks were only rubbed on with the finger, so that the patches were not exactly uniform, especially the lightest one (1). In this case



Curves showing the absorptions of the same ink printed to different depths.

the readings were checked by drawing a smooth curve between the points found, and the numbers taken from the curve, where they were

obviously wrong. Otherwise the actual means were used so that the error might be the more readily estimated. Had I been able to obtain printed uniform patches I have no doubt the results would have been better. But even these can leave no doubt of the approximate truth of the law.

A Blue Ink from Winstone.

Curve No. 20.—I. A very faint rubbing.

Scale.	Aperture.	Average.	Corrected from curve.	White.	Per cent.	Log of ratio.
10	10·0 — 10·5	10·2	—	81·2	12·4	−0·906
15	16·0 — 14·5	15·2	—	85·4	17·7	−0·752
20	27·0 — 27·0	27·0	—	87·0	31·0	−0·508
25	35·5 — 35·0	35·2	—	86·6	40·6	−0·392
30	43·0 — 43·0	43·0	—	83·5	51·6	−0·288
35	43·0 46·0 45·0	44·6	—	77·2	57·8	−0·238
40	45·0 — 46·0	45·5	—	69·3	65·7	−0·182
45	50·0 53·0 48·50	50·0	—	64·4	70·0	−0·155
50	38·0 42·0 39·0	39·6	—	57·3	69·1	−0·160
55	38·5 — 38·0	38·2	—	54·7	69·7	−0·157

Curve No. 21.—II. A little darker than I.

Scale.	Aperture.	Average.	Corrected from curve.	White.	Per cent.	Log of ratio.
10	4·5 — 4·0	4·2	—	81·2	5·2	−1·384
15	6·0 — 6·5	6·7	—	85·4	7·8	−1·107
20	10·5 — 9·5	10·0	—	87·0	11·5	−0·939
25	18·0 — 16·5	17·2	—	86·6	20·0	−0·699
30	23·0 — 24·0	23·5	—	83·5	28·1	−0·551
35	27·5 26·5 28·0	27·3	—	77·2	35·5	−0·450
40	32·0 — 32·0	32·0	—	69·3	46·0	−0·337
45	33·0 35·0 33·0	33·6	—	64·4	52·2	−0·282
50	27·0 28·0 32·0	29·0	32	57·3	55·8	−0·254
55	30·0 29·5 —	29·8	—	54·7	53·4	−0·272

Curve No. 22.—III. Still darker rubbing.

Scale.	Aperture.	Average.	Corrected.	White.	Per cent.	Log of ratio.
10	3·5 — 3·2	3·3	—	81·2	4·07	-1·391
15	4·0 — 4·0	4·0	—	85·4	4·68	-1·329
20	7·5 — 8·0	7·8	—	87·0	9·07	-1·042
25	13·5 — 13·5	13·5	—	86·6	15·6	-0·806
30	19·0 18·2 20·0	19·0	—	83·5	22·8	-0·642
35	23·0 — 23·2	23·1	—	77·2	29·9	-0·524
40	26·0 28·0 27·0	27·0	—	69·3	38·9	-0·410
	28·5 — 26·5					
45	3·1 — 28·5	29·0	—	64·4	45·0	-0·346
50	28·0 — 28·0	21·0	—	57·3	48·8	-0·312
55	21·0 — 22·0	21·5	—	54·7	39·3	-0·406

Curve No. 23.—IV. Very heavy rubbing of the same ink.

Scale.	Aperture.	Average.	Average corrected from curve.	White.	Per cent.	Log of ratio.
10	1·5 — 1·5	1·5	1·5	81·2	1·85	-1·732
15	2·0 — 2·0	2·0	1·9	85·4	2·12	-1·674
20	2·0 — 2·0	2·0	2·1	87·0	2·42	-1·616
25	1·5 — 1·7	1·6	2·2	86·6	2·55	-1·594
30	2·5 — 2·3	2·4	—	83·5	2·88	-1·541
35	3·8 — 4·0	3·9	—	77·2	5·06	-1·296
40	6·1 6·6 6·2	6·5	—	69·3	9·37	-1·028
45	8·5 — 8·8	8·6	—	64·4	13·35	-0·874
50	8·2 — 7·5	7·9	—	57·3	13·77	-0·861
55	6·5 — 6·5	6·5	—	54·7	11·85	-0·926

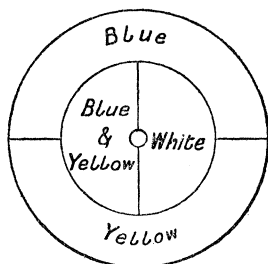
Scale.	III I Ratio $\frac{e}{f}$.	IV III Ratio.	III II Ratio.
10	—	—	—
15	(1·77)	—	(1·00)
20	2·05	—	1·11
25	2·05	(1·98)§	1·15
30	2·24	2·41	1·16
35	2·20	2·42	1·16
40	2·27	2·51	1·21
45	2·26	2·54	1·23
50	1·95*	2·76*	1·23§
55	(2·61)	2·27	(1·48)
Mean	2·16	2·45	1·18

§ These two are the only ones which have been corrected from the curve.

These ratios (omitting the ones in brackets where the lack of light made the readings too uncertain) do not differ from the mean by one in eight, and omitting the ones marked *, the difference is not more than one in fourteen.

20. *Experiment to see whether the Colour produced by Blue and Yellow side by side matched that when they were printed on one another.*

Mr. Gamble prepared for me a card printed with blue and one printed with yellow. Then also a third printed with both the blue and the yellow; this, of course, made green. I cut the blue and yellow into discs about 8 inches in diameter, and the green into one about



4 inches in diameter. Also I cut a white—the white on which they had all been printed—into a disc 4 inches in diameter. Then I set them on a motor, as in Maxwell's discs, with equal areas of blue and yellow, and of green and white. These ought when rotated to match if the inks are to be used for process work. I found there was a very great difference in both hue and luminosity. To make a match it was necessary to reduce the yellow to only 125° and increase the blue to 235° , and by making the white 125° and the green 235° the match was correct in luminosity as well as hue.

Thus, as would be expected since the inks had not abrupt absorptions, it made a very great difference if the colours were superposed or were side by side.